# Comment to the paper " The energy conservation law for electromagnetic field in application to problems of radiation of moving particles "

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In the paper [1] the energy conservation law (the Poynting theorem) was applied to a problem of radiation of a charged particle in an external electromagnetic field. The authors consecutively and mathematically strictly solved the problem but received wrong result. They derived an expression which includes a change of the energies of the electromagnetic fields accompanying the homogeneously moving particle $\Delta W = W_2 - W_1$ corresponding to the initial and final velocity of the particle (see expression (19) in [1]). The energy of the field accompanying the particle $W$ is the energy of the particle of the electromagnetic origin. It should not enter the solution of the problem. The authors do not specify the dimensions of the particle. For pointlike particle this energy and the change $\Delta W$ are infinite values and consequently the expression (19) loses sense. In quantum theory the derived expression require some renormalization. In classical theory in the section devoted to the energy conservation law the energy of the accompanying field that is the energy of the particle of the electromagnetic origin is hidden in the total energy of the electromagnetic origin and hence it is appeared unnoticed. For this reason the solutions based on the use of the energy conservation law lead to the wrong results when $\Delta W \neq 0$ [2]. The received solutions differ from the solutions based on the equations of motion of particles in the external fields.

We will explain our observation using the second example considered by the authors. This example was formulated as follows. Let a charged particle is moving in a positive direction of the axis "z" with a velocity $\vec{v}_1$. In some area with the linear dimensions $L$ located near to the origin of the reference frame the external electromagnetic fields are created where the velocity of the particle is varied under some law which is not specified. Then the particle go out from this area and it's velocity accepts the value $\vec{v}_2$ which hereinafter is not changed. The authors proceed from the expression for the energy conservation law of the form

$$\frac{\partial}{\partial t}\int_V \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} dV = -\int_V \vec{\jmath}\vec{E} dV - \frac{C}{4\pi}\int_S [\vec{E}\vec{H}] d\vec{S}, \tag{1}$$

where $\vec{E}$, $\vec{H}$ are vectors of the electric and magnetic fields respectively created in a general case by a set of particles, charged bodies and magnets, $\vec{\jmath}$ a vector of density of a current, the sign $V$ under the integral means that the integral is carried out through a chosen volume $V$ and the sign $S$ means that the integral is carried out through a surface $S$ limiting this volume. This law (the Poynting theorem) follows from the Maxwell's equations.

From this law the authors came to the expression

$$\frac{c}{4\pi}\int_{-\infty}^{+\infty} dt \int_S [\vec{E}"\vec{H}"] d\vec{S} = -\int_{-\infty}^{+\infty} dt \int_V \vec{\jmath}\vec{E} dV - \Delta W, \tag{2}$$

where the vectors $\vec{E}$", $\vec{H}$" are vectors of free electric and magnetic fields emitted by a particle, $W_1 = (1/8\pi)\int(|\vec{E}_1|^2 + |\vec{H}_1|^2)dV$, $W_2 = (1/8\pi)\int(|\vec{E}_2|^2 + |\vec{H}_2|^2)dV$ are the total energies of the electromagnetic fields, created by the homogeneously moving charged particle in the unlimited space (the energies of the accompanying field), vectors $\vec{E}_1$, $\vec{H}_1$ and $\vec{E}_2$, $\vec{H}_2$ are the vectors of the electric and magnetic field strengths created by a particle moving homogeneously with velocities $\vec{v}_1$, $\vec{v}_2$ accordingly. It is supposed that the boundary of the volume $V$ is chosen so far that the wavepacket of radiation was in time to be separated from the field of the charged particle so that free fields of radiation and the field accompanying the particle are not overlapped.

Further the authors go to the conclusion that the flow of radiation from the volume $V$ according to (2) is determined not only by the work of forces acting on the charged particles by fields (integral from $\vec{j}\vec{E}$) but also by change of the energy of the accompanying electromagnetic field $\Delta W$.

Now we notice that the vector of the electric field strength in the region of location of the particle can be presented in the form $\vec{E} = \vec{E}_{ext} + \vec{E}_s$, where $\vec{E}_{ext}$ is the vector of the external electric field strength created by charged bodies and other particles, $\vec{E}_s$ vector of the electric field strength produced by the particle under consideration. Therefore the external fields and the fields produced by a particle (inertial and radiating self-fields) were took into account in the change of the energy of the particle $\varepsilon$ and the value $\int_V \vec{j}\vec{E}dV = d\varepsilon/dt$ [3], [4]. That is why the value $\int_{-\infty}^{+\infty} dt \int_V \vec{j}\vec{E}dV = \Delta\varepsilon = \varepsilon_2 - \varepsilon_1 = mc^2(\gamma_2 - \gamma_1)$ in the equation (2) is the change of the total energy of the particle where $m$ is the weight of the particle, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = |\vec{v}/c|$, the subscripts $1, 2$ are related to initial and final velocity of the particle. The value $c/4\pi \int_{-\infty}^{+\infty} dt \int_S [\vec{E}"\vec{H}"]d\vec{S} = \varepsilon^{rad}$ is the energy of the electromagnetic radiation emitted by the particle in the form of free electromagnetic waves. Thus the expression (2) can be presented in the form

$$\Delta\varepsilon = -\varepsilon^{rad} - \Delta W. \tag{3}$$

In the presented example it was supposed that the external fields are static and the energy of these fields is constant. In a static case the electric field is potential one $\int \vec{E}_{ext}(\vec{r})d\vec{r} = 0$. The external field could be the magnetic one. Therefore obviously the change of the energy of the particle in the case of static fields should be equal to the energy of radiation taken with the negative sign

$$\Delta\varepsilon = -\varepsilon^{rad}. \tag{4}$$

The expression (4) follows also from the equations of motion of the particle in the external fields taking into account the radiation reaction force and the laws of radiation of a particle in the external fields which are determine the rate of losses of the energy of the particle in the form of radiation.

Contrary to expected result a superfluous term has appeared in the expression (3) which is equal to the change of the energy of the field $\Delta W$ accompanying the particle and differ from zero as the initial $v_1 = |\vec{v}_1|$ and final $v_2 = |\vec{v}_2|$ velocities are not equal $(v_1 - v_2 \neq 0)$.

The presence of the superfluous term $\Delta W$ is in accordance with the conclusions made in the paper [2] that from the equations of Maxwell-Lorentz does not follow the correct energy conservation law that is the law which describe the nature correct way since the equations of Maxwell and equations of Lorentz are inconsistent. The expression (1) contains a logic error consisting in the fact that in the first field term of this expression the energy of the electromagnetic field is included and in this energy the energy of the accepted electromagnetic field of the particle i.e. the energy of the particle of electromagnetic origins is hidden. It means that the energy of the particle of the electromagnetic origin in the equation (1) is presented in two terms (left and first right term). Accordingly the energy of the particle of the electromagnetic origin in the equation (3) is also presented in two terms ($\Delta\varepsilon$ and $\Delta W$). We should like to remind that energy of the particle is a sum of energies of the electromagnetic and non-electromagnetic origin and in the case of pointlike particles they are infinite and have opposite sign and their sum presents the experimentally observable value $\varepsilon$ [3]. Thus the energy of particles of the electromagnetic origin is presented in expressions (1), (3) twice and that is why the Poynting theorem generalized on a case of a system of fields and particles becomes incorrect. In the case of pointlike particles the value $\Delta W$ in the expression (3) is infinite when the value $v_1 - v_2 \neq 0$ and that is why this expression loses its sense. The logic error consist in the double inclusion of the energy of the particle of the electromagnetic origin in the same equation.

We would like to remind the energy conservation law for a system of the electromagnetic field and particles in an integral form $\partial\varepsilon^{\Sigma}/\partial t = 0$ or $\varepsilon^{\Sigma} = const$ where

$$\varepsilon^{\Sigma} = \int \frac{|\vec{E}|^2 + |\vec{H}|^2}{8\pi} dV + \Sigma\varepsilon_i, \tag{5}$$

$\varepsilon_i$ is the energy of a particle $i$, and the integration is carried out through the whole space [3]. The first term from the right in the expression (5) contains both the free field of radiation emitted by charged particles and the field accompanying these particles. The dimensions, charge, and weight of the particle and also their structure can be arbitrary. That is why massive charged bodies and magnets can enter in (5). Massive bodies can have complex structure. In this case the exchange of the energy of electromagnetic fields is possible in internal degrees of freedom of a body (for example, in heating the body). At that the weight and, accordingly, energy of bodies $\varepsilon_i$ will be increased.

In a general case if the wave packet of radiation emitted by a system of particles will be in time to be separated from the fields accompanying these particles then the change of the energy of the system of particles $\Delta\varepsilon = \Sigma\Delta\varepsilon_i$ and the change of the energy of the electromagnetic field according to (5) will be determined by the same expression (3) where now $\Delta W$ is the change of the energy of the accompanying electromagnetic fields of all particles[1]. It means that the conclusion about a logic mistake made at the proof of the energy conservation law for a system of electromagnetic field and particles made in the paper [2] for a general case non-obviously was confirmed by the authors of the commented paper in their example.

[1]Certainly it is possible to receive this conclusion proceeding from the expression (1).

At the derivation of the energy conservation law for the system of the electromagnetic field and particles a mistake was made which further was accepted by repetition in many papers and textbooks. Therefore the interpretation of this law in the textbooks should be changed and should be treated in the form of an open question in classical electrodynamics.

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# References


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